Can high-velocity stars reveal black holes in globular clusters?

G.A. Drukier & C.D. Bailyn

Dept. of Astronomy, Yale University, P.O. Box 208101, New Haven, CT, 06520-8101, USA

`drukier, bailyn@astro.yale.edu`

ABSTRACT

We estimate the number of individual, fast-moving stars observable in globular clusters under the assumption that the clusters contain massive central black holes which follow the galactic $M_{\rm BH} - \sigma$ relationship. We find that radial velocity measurements are unlikely to detect such stars, but that proper motion studies could reveal such stars, if they exist, in the most likely clusters. Thus, *HST* proper motion studies can test this hypothesis in a few nearby clusters.

Subject headings: globular clusters: general — stellar dynamics

1. Introduction

It has been suggested (Gebhardt, Rich, & Ho 2002), that globular clusters may contain central black holes with masses lying along the extension of the correlation between black hole mass and bulge velocity dispersion as seen for galaxies (Ferrarese & Merritt 2000; Gebhardt et al. 2000). However it has also been suggested that the observational evidence can be otherwise explained (Baumgardt et al. 2003a,b; Dull et al. 2003). These difficulties of interpretation are compounded by the small number of observable stars in the cores of globular clusters. In contrast to a galactic center, current observations approach the limit when the uncertainty of an aggregate measurement, such as a velocity dispersion, cannot be reduced by further observations. Once every star has been observed, there is nothing more to be done. (See Drukier et al. 2003 for a general discussion of issues relating to the observation and interpretation of velocity dispersions in globular clusters.) This is particularly the case for central black holes, since there is only a relatively small region over which the black hole provides the dominant contribution to the gravitational potential.

Most current observational work relating to the possible presence of central black holes in globular clusters has been done using radial velocity measurements to determine the central value and radial gradient of the velocity dispersion. In this *Letter* we explore an

alternative approach, namely the use of proper motion measurements to identify individual stars whose velocities are strongly influenced by the presence of a central black hole. We find that existing crude models suggest that this approach may be fruitful, and that the considerable theoretical and observational work that will be required to apply this approach in practice may thus be justified. In §2, we estimate the number of stars with anomalously high velocities caused by a central black hole, and in §3 we generate a list of the clusters with the highest probability of observing such stars. We find that for plausible assumptions such stars might well be observable with HST in the most favorable clusters.

2. Estimating the number of high-velocity stars

Consider a globular cluster with central velocity dispersion σ_0, where this is the dispersion outside the radius of influence, r_h, of a black hole with mass $M_\bullet$, given by

$$r_h \equiv \frac{GM_\bullet}{\sigma_0^2}. \tag{1}$$

We define $\sigma_\bullet(r)$ as the one-dimensional velocity dispersion and $\Sigma(r)$ as the stellar surface density within r_h. Outside r_h, the influence of the black hole is too small to significantly affect stellar orbits. For simplicity, we assume that the velocities are distributed as a Gaussian with dispersion $\sigma_\bullet(r)$. Now, at a radius r, the fraction of stars which will have velocities greater than some multiple k of the velocity dispersion σ_0 is given by

$$f(r) = \frac{2}{\sqrt{2\pi}\sigma_\bullet(r)} \int_{k\sigma_0}^{\infty} \exp\left(-\frac{v^2}{2D\sigma_\bullet^2(r)}\right) dv, \tag{2}$$

where D is the dimensionality of the observed velocity (1 for radial velocities, 2 for proper motions, and 3 for space velocities). While the upper limit could, in principle, be some multiple of the escape velocity, this is difficult to define without detailed modeling of the structure of the whole cluster. Instead, as the complementary error function drops off rapidly with increasing argument, we will just use the infinite upper limit shown. This will lead to a slight overestimate in the number of expected stars, but this effect is small compared to that due to the uncertainty in β discussed below. With this limit, equation (2) reduces to

$$f(r) = \sqrt{D}\,\mathrm{erfc}\left(\frac{k\sigma_0}{\sqrt{2D}\sigma_\bullet(r)}\right). \tag{3}$$

The total number of stars with "significantly" high velocities (significant being defined here as k times the velocity dispersion σ_0 outside r_h) is then given by

$$N = 2\pi \int_0^{r_h} r\Sigma(r) f(r) dr. \tag{4}$$

The only existing dynamical investigation of the effects of a central black hole on the structure of a globular cluster is that of Cohn & Kulsrud (1978) who used the Fokker-Planck equation to integrate a steady-state, anisotropic distribution function in the vicinity of a black hole. They found that the projected velocity dispersion and surface density profiles could be approximated, for $r < r_h$, by

$$\sigma_\bullet^2(r) = \begin{cases} 0.4\sigma_0^2 \frac{r_h}{r} & r < 0.4 r_h \\ \sigma_0^2 & 0.4 r_h \le r \le r_h \end{cases}, \tag{5}$$

$$\Sigma(r) = \Sigma_0 \left(\frac{r_h}{r}\right)^{0.5}. \tag{6}$$

The two terms in equation (5) deal with the flattening of the velocity dispersion profile near r_h. Substituting these into equation (4) leads, after a suitable rescaling of the integration variable, to

$$N = 2\pi \Sigma_0 D^2 r_h^2 I_\bullet(k, D), \tag{7}$$

where

$$I_\bullet(k, D) = \int_0^{0.4/D} \sqrt{x}\ \mathrm{erfc}\left(k\sqrt{\frac{x}{0.8}}\right) dx + \mathrm{erfc}\left(\frac{k}{\sqrt{2D}}\right) \int_{0.4/D}^{1/D} \sqrt{x} dx. \tag{8}$$

The integral can be evaluated numerically and gives $I_\bullet(3, \{1, 2, 3\}) = \{1.1, 1.5, 1.5\} \times 10^{-2}$ to two figures. By way of comparison, if no black hole is present then, assuming $\sigma_\bullet(r) = \sigma_0$ and $\Sigma(r) = \Sigma_0$,

$$I_0(k, D) = \sqrt{D}\ \mathrm{erfc}\left(\frac{k}{\sqrt{2D}}\right) \int_0^{1/D} x dx, \tag{9}$$

for which $I_0(3, \{1, 2, 3\}) = \{1.4, 6.0, 8.0\} \times 10^{-3}$, and the expected number of stars is given by equation (7) with I_0 in place of $I_\bullet$.

The galactic black hole mass versus velocity dispersion correlation (Ferrarese & Merritt 2000; Gebhardt et al. 2000) is usually given in the form

$$M_\bullet = 10^\alpha \left(\frac{\sigma_0}{\sigma_*}\right)^\beta M_\odot, \tag{10}$$

where σ_0 is a suitable velocity dispersion and $\sigma_* = 200$ km s^{-1}. The uncertainty in the power-law slope is the largest source of uncertainty in our estimated number of high-velocity stars. Recent estimates for β (the variation in α is small since all studies agree for $\sigma_0 \sim \sigma_*$) range from $\beta^L = 4.02 \pm 0.32$ ($\alpha^L = 8.13 \pm 0.06$) (Tremaine et al. 2002) to $\beta^H = 4.65 \pm 0.48$ ($\alpha^H = 8.17 \pm 0.07$) (Merritt & Ferrarese 2001). These agree within their quoted errors, but, on extrapolation to the globular cluster regime, the use of β^L predicts 20 times as many high-velocity stars as does the use of β^H. We therefore give results for both these slopes below. The original claim for globular clusters (Gebhardt, Rich, & Ho 2002) used β^L.

Substituting equations (1) and (10) into equation (7) we arrive at the following

$$N = 2\pi G^2 10^{2\alpha} \sigma_*^{-2\beta} I_\bullet(k, D) D^2 \Sigma_0 \sigma_0^{2(\beta-2)}, \tag{11}$$

where σ_0 is measured in km s^{-1} and Σ_0 is the number of *measurable* stars per square parsec. What we can easily observe is not Σ_0 but the central surface density, μ_0, so we need to rewrite Σ_0 in terms of μ_0 for a reasonable globular cluster stellar population. For a defined population of stars, let $\bar{L}_*$ be the cluster luminosity per star in that population in solar units, and let g_* be the fraction of these stars that are usefully measurable. Then for μ_0 in V magnitude per square arc second, and taking $M_{V\odot} = 4.79$

$$\Sigma_0 = 10^{-0.4(\mu_0 - 26.37)} g_* \bar{L}_*^{-1}. \tag{12}$$

In this case

$$N = I_\bullet(k, D) D^2 10^{-0.4\mu_0} g_* \bar{L}_*^{-1} \hat{\alpha} \sigma_0^{\hat{\beta}}, \tag{13}$$

where $\hat{\alpha}^L = 2.37 \times 10^4$, $\hat{\beta}^L = 4.04$, $\hat{\alpha}^H = 36.0$, and $\hat{\beta}^H = 5.30$.

For our problem, $\bar{L}_*$ is determined by the luminosity function in the core of the cluster in question. It needs to take into account mass segregation effects and other possible population peculiarities as might be indicated by, for example, color gradients. The measurable fraction, g_*, is an observational selection effect on the luminosity function. It will depend on the observational technique to be used, the distance of the cluster, crowding and so forth.

Since a complete stellar census is a difficult undertaking, we estimate $\bar{L}_*$ as follows. Define our population to be all the stars brighter than some magnitude, V_d (e.g. the expected magnitude limit of the observations) in a cluster color-magnitude diagram (CMD). Then,

$$\bar{L}_* \approx \bar{L}_b + f^{-1} \bar{L}_f. \tag{14}$$

The quantities $\bar{L}_b$ and $\bar{L}_f$ are the mean luminosities for the bright and faint parts of the CMD (divided at V_d) and f is the ratio of the number of stars brighter than V_d to the number of those fainter. $\bar{L}_b$ can be estimated directly from the cluster CMD, for which purpose we use the CMDs in the compilation by Piotto et al. (2002). $\bar{L}_f$ and f can be estimated from the corresponding theoretical luminosity function (LF). We use those from the models of Silvestri et al. (1998). One limitation is that these LFs only include stars from the hydrogen burning limit to the tip of the red giant branch, so we must use the CMD to correct the ratio of the number of stars in each part of the LF, $f_{\rm LF}$, for the contribution of post-RGB stars. If, brighter than V_d, we see in the CMD n_R stars in the regions covered by the luminosity function, and n_B more evolved stars elsewhere, then

$$f = f_{\rm LF} \left(1 + \frac{n_B}{n_R}\right). \tag{15}$$

By substituting equations (15) and (14) into equation (13), we can now estimate N for any given cluster, subject to the selection of g_*. We proceed to do this in the next section.

3. Best Target Clusters

We present in Table 1 some relevant numbers for the 12 clusters most likely to show evidence for a central black hole. All other clusters are estimated to have at least a factor of three fewer observable high-velocity stars than NGC 5824. The value of r_h in the fifth column of the table is calculated assuming α^L and β^L, and is given in arc seconds for the distance in column 4. Note that while this radius scales as $\sigma_0^{\beta-2}$, the radii for β^H are smaller by a factor of approximately 5.

Our estimates of $\bar{L}_*$ are given in the sixth column of Table 1. The Piotto et al. (2002) CMDs cover 9 of our listed clusters. For those not listed we have used $\bar{L}_* = 11.4$, the mean for the other 9. These are marked by a colon after $\bar{L}_*$. We have taken V_d for each cluster such that the division is at $M_V = 4.5$, a magnitude or so below the turn-off for these low-metallicity systems. For the luminosity functions we use a very flat mass-spectral-index x of -0.5, where the Salpeter value is 1.35. Mass segregation is likely to have removed most of the low-mass stars from the cluster center—many of the mass functions are actually inverted (see e.g. Sosin 1997; de Marchi, Paresce, & Pulone 2000; Albrow, De Marchi, & Sahu 2002)—so this should be a reasonable approximation. In any case, the luminosity-function-dependent term $f^{-1}\bar{L}_f$ in equation (14) is roughly 10% that of $\bar{L}_b$, so the estimate of $\bar{L}_*$ is dominated by the stars in the CMD.

The final two columns give the expected number of stars for the two estimates of the $M_{\rm BH} - \sigma$ relation relative to the number expected for M 15 (NGC 7078), the cluster with the best studied core. Only two clusters, NGC 6388 and NGC 6441, are likely to have more fast-moving stars than M 15 for the lower slope. NGC 5139 (ω Cen) would have roughly the same number as M 15 using the higher slope, but the absolute numbers are down by a factor of 24 with respect to the low slope. To complete our estimate, it remains only to specify g_* to get the number of fast-moving stars we could detect. We consider two scenarios: radial velocities and proper motions.

For radial velocity measurements, g_* is likely to be small, as only the brightest, least crowded, central stars are suitable. For the case of M 15, if we compare the number of stars with radial velocities as compiled by Gerssen et al. (2002) with the *HST* photometric list in van der Marel et al. (2002) then, with $r_h = 1.''3$, we find that about 4% of the stars brighter than V_d have velocities. Within $r_h/2$ this is 7%, but within $r_h/4$, where most of the

Table 1. Parameters for the most likely clusters

Cluster	μ_0 [a]	σ_0 [b]	d [c]	r_h [d]	$\bar{L}_*$	$\frac{N}{N^L_{\rm M15}}$	$\frac{N}{N^H_{\rm M15}}$
NGC 6388	14.55	18.9	10.0	2.5	11.4	2.5	3.7
NGC 6441	14.99	18.0	11.7	2.0	11.3	1.4	1.9
NGC 7078	14.21	14.0	10.3	1.3	11.8	1.0	1.0
NGC 6715	14.82	14.2	26.8	0.5	11.4:	0.6	0.6
NGC 5139	16.77	22.0	5.3	6.5	11.4:	0.6	1.1
NGC 104	14.43	11.5	4.5	2.1	9.8	0.4	0.3
NGC 6266	15.35	14.3	6.9	2.1	10.5	0.4	0.4
NGC 2808	15.17	13.4	9.6	1.3	10.7	0.4	0.4
NGC 1851	14.15	10.4	12.1	0.6	11.2	0.3	0.2
NGC 6752	15.20	12.5	4.0	2.8	11.4:	0.3	0.2
NGC 6093	15.19	12.4	10.0	1.1	12.5	0.2	0.2
NGC 5824	15.08	11.6	32.0	0.3	13.7	0.2	0.1

[a]Central V surface brightness taken from the 2003 Feb revision of the Harris (1996) compilation.

[b]Central velocity dispersion in km s^{-1} from Pryor & Meylan (1993) with the exceptions of NGC 7078 (Gerssen et al. 2002), NGC 6752 (Drukier et al. 2003), and NGC 5139 (Merritt, Meylan, Mayor 1997).

[c]Distance in kpc from Harris (1996).

[d]Black hole region of influence as defined by equation (1) in arc seconds, assuming α^L and β^L.

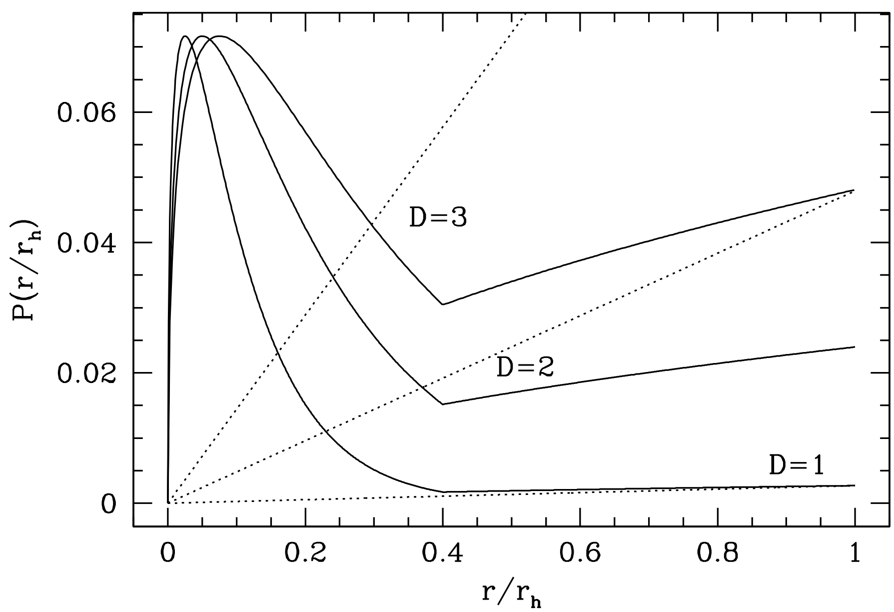

Fig. 1.— The expected radial distribution of high velocity stars with (solid) and without (dotted) a black hole in the cluster center for the case $k = 3$ and $D = 1$, 2, and 3. These distributions are related to the integrands in equations (8) and equation (9) through a coordinate transformation of the radial variable. The break at $0.4r_h$ is due to the break in equation (5).

fast-moving stars are to be found, there are no velocities out of the 10 stars available. Thus, for radial velocities in M 15, $g_* = 0.07$ at best, and we would expect to see $N^L_{\rm M15} = 0.1$ stars, moving with a relative radial velocity $\geq 3\sigma_0$ if a black hole is present. That no such star has been seen is, therefore, unsurprising and uninformative. Combining the entire dozen most-likely clusters yields perhaps one or two fast-moving stars. $N^H_{\rm M15} = 0.004$ stars. The number expected is effectively zero.

A further problem with using radial velocities to detect the fast-moving stars lies in their radial distribution as given by the integrands in equations (8) and (9) after an appropriate coordinate transformation. We compare these in Figure 1 for $k = 3$ and the three values of D. In all three cases, the fast-moving stars are concentrated well within r_h. The break point at $0.4r_h$ is the result of the break in $\sigma_\bullet$ at this radius. Note the very different expected radial distributions for the fast-moving stars depending on whether (*solid*) or not (*dotted*) there is a black hole. If there is a black hole, the fast-moving stars should be concentrated in the center of the cluster, with a maximum inside $0.1r_h$. For radial velocities, the high-velocity stars are confined within about $0.15r_h$ or $0\farcs3$ for the nearer clusters. For the higher-dimensional

velocities, the detection region is somewhat broader. Clearly, radial velocities are unlikely to prove the question one way or another.

For proper motions, the situation is much better, at least for nearby clusters for which such observations are feasible. In the case of NGC 6752, Rubenstein & Bailyn (1997) measure 153 stars brighter than V_d within $r_h = 2\overset{\prime\prime}{.}6$ of the cluster center. To the same radius, Drukier et al. (2003), measure the proper motions of 15 stars. Their detection region only covers about a third of the area within r_h due to gaps in their 1999 data. These gaps, unfortunately, covers much of the very center of the cluster, so it is difficult to estimate the detection rate within about $0.3r_h$ where we expect to find most of the fast-moving stars. Crowding is bound to be a problem in this region, even for *HST*. Extrapolating from the most crowded regions studied by Drukier et al. (2003) and taking into account the gaps in their coverage, we estimate that $g_* = 0.2$ is appropriate for proper motions. In this case $N^L_{\rm M15} = 2$ stars. If this $M_{\rm BH} - \sigma$ conjecture is correct, the top three clusters should contain of order 10 high-velocity stars between them with the balance of the 12 clusters in Table 1 contributing another 5 or so in total. In addition, their radial range is double that for radial velocities. On the other hand, $N^H_{\rm M15} = 0.08$ stars, and the black holes will be undetectable by this method.

Proper motions have the feature that their uncertainty is inversely proportional to the time baseline and proportional to the distance. Nonetheless, the top three clusters, all with distances between 10 and 12 kpc, should be close enough that, given sufficient data, the black hole hypothesis can be tested, assuming a value of β at the lower end of its range. The main limitation at this distance will be the increase in effective crowding, proportional to the distance squared. For the top three clusters in Table 1, crowding will be 10 to 16 times higher than in the NGC 6752 observations of Drukier et al. (2003).

In the absence of a black hole, the flat extrapolation would lead us to expect to see one or two fast-moving stars within r_h in each of our top three candidates. These stars will have a radial distribution proportional to radius, and should be found in the vicinity of r_h, not concentrated within $0.2r_h$ as is predicted under the black hole conjecture. Fast-moving stars can have origins other than a black hole, of course. Ejection from the core during core-collapse is one plausible mechanism for producing such stars (Drukier et al. 1999), but their velocity vectors will be radial unlike a star in orbit around a black hole.

We caution that the numbers presented here are only estimates and depend on the scalings found by Cohn & Kulsrud (1978). Those models are single-mass anisotropic Fokker-Planck simulations for the steady-state stellar distribution in the vicinity of the black hole. More modern models, which should include, at the very least, a range of stellar masses and a self-consistent potential, will be needed to fully assess the significance of any fast-moving stars which are observed in these clusters. The estimates made here also depend on the

current central velocity dispersions in the globular clusters. Since globular clusters can lose a large fraction of their mass due to stellar evolution and stellar-dynamical evolution, the numbers provided in Table 1 may well be underestimated if the proper velocity dispersion to use in determining black hole masses is the original value, not the current one. Further, the mass of any central black hole will have increased to some extent due to the capture of cluster stars. Using double the current velocity dispersion, for example, would increase $N^L_{\rm M15}$ by a factor of 16 and $N^H_{\rm M15}$ by a factor of 39, in which case the higher slope also predicts significant numbers of observable stars in the top few clusters. Given the sensitivity to these effects, obtaining reliable estimates for the numbers of high-velocity stars will require fully evolving models. Even in the event that proper-motion studies uncover no fast moving stars, such models will allow for firm upper limits on the mass of any black hole. Constructing such models, and carrying out the necessary proper motion observations, is no small task, but given the strong interest and controversy currently surrounding this topic, we believe that efforts along these lines should be vigorously pursued.

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